

8. Berechne ohne Taschenrechner!

$$(a) \quad \sqrt[3]{4} \cdot \sqrt[3]{7} = \sqrt[3]{4 \cdot 7} = \sqrt[3]{28}$$

$$(c) \quad (\sqrt{5})^6 = \left(5^{\frac{1}{2}}\right)^6 = 5^{\frac{1}{2} \cdot 6} = 5^3$$

$$(e) \quad \sqrt{\sqrt{3}} = \left(3^{\frac{1}{2}}\right)^{\frac{1}{2}} = 3^{\frac{1}{2} \cdot \frac{1}{2}} = 3^{\frac{1}{4}} = \sqrt[4]{3}$$

$$(g) \quad \sqrt{32} \cdot \sqrt[5]{16} = \left(2^5\right)^{\frac{1}{2}} \cdot \left(2^4\right)^{\frac{1}{5}} = 2^{5 \cdot \frac{1}{2}} \cdot 2^{4 \cdot \frac{1}{5}} = \\ = 2^{\frac{5}{2}} \cdot 2^{\frac{4}{5}} = 2^{\frac{5}{2} + \frac{4}{5}} = 2^{\frac{25}{10} + \frac{8}{10}} = \\ = 2^{\frac{33}{10}} = 2^{\frac{30}{10}} \cdot 2^{\frac{3}{10}} = 2^3 \cdot \sqrt[10]{2^3}$$

$$(i) \quad \frac{\sqrt{12}}{\sqrt[3]{3}} = \frac{\left(3 \cdot 2^2\right)^{\frac{1}{2}}}{3^{\frac{1}{3}}} = \frac{3^{\frac{1}{2}} \cdot 2^{\frac{2}{2}}}{3^{\frac{1}{3}}} = 2 \cdot 3^{\frac{1}{2} - \frac{1}{3}} = \\ = 2 \cdot 3^{\frac{3}{6} - \frac{2}{6}} = 2 \cdot 3^{\frac{1}{6}} = 2 \cdot \sqrt[6]{3}$$

$$(b) \quad \frac{\sqrt[4]{5}}{\sqrt[4]{2}} = \sqrt[4]{\frac{5}{2}}$$

$$(d) \quad \left(\sqrt[3]{4}\right)^{-2} = \left(\left(2^2\right)^{\frac{1}{3}}\right)^{-2} = 2^{2 \cdot \frac{1}{3} \cdot (-2)} = 2^{-\frac{4}{3}} = \\ = \frac{1}{2^{\frac{4}{3}}} = \frac{1}{2^{\frac{3}{3}} \cdot 2^{\frac{1}{3}}} = \frac{1}{2 \cdot \sqrt[3]{2}}$$

$$(f) \quad \sqrt[4]{\sqrt[3]{2}} = \left(2^{\frac{1}{3}}\right)^{\frac{1}{4}} = 2^{\frac{1}{3} \cdot \frac{1}{4}} = 2^{\frac{1}{12}} = \sqrt[12]{2}$$

$$(h) \quad \sqrt[4]{5} \cdot \sqrt[3]{25} = 5^{\frac{1}{4}} \cdot \left(5^2\right)^{\frac{1}{3}} = 5^{\frac{1}{4}} \cdot 5^{2 \cdot \frac{1}{3}} = 5^{\frac{1}{4}} \cdot 5^{\frac{2}{3}} = \\ = 5^{\frac{1}{4} + \frac{2}{3}} = 5^{\frac{3}{12} + \frac{8}{12}} = 5^{\frac{11}{12}} = \sqrt[12]{5^{11}}$$

$$(j) \quad \frac{\sqrt[6]{45}}{\sqrt[4]{5}} = \frac{\left(3^2 \cdot 5\right)^{\frac{1}{6}}}{5^{\frac{1}{4}}} = \frac{3^{\frac{2}{6}} \cdot 5^{\frac{1}{6}}}{5^{\frac{1}{4}}} = 3^{\frac{1}{3}} \cdot 5^{\frac{1}{6} - \frac{1}{4}} = \\ = 3^{\frac{1}{3}} \cdot 5^{\frac{2}{12} - \frac{3}{12}} = 3^{\frac{1}{3}} \cdot 5^{-\frac{1}{12}} = \frac{\sqrt[3]{3}}{\sqrt[12]{5}}$$

9. Ziehe teilweise die Wurzel!

$$\sqrt{80} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5} = \sqrt{2^2 \cdot 2^2 \cdot 5} =$$

$$(a) \quad = 2 \cdot 2 \cdot \sqrt{5} = 4 \cdot \sqrt{5}$$

$$(c) \quad \sqrt[3]{81} = \sqrt[3]{3 \cdot 3 \cdot 3 \cdot 3} = \sqrt[3]{3^3 \cdot 3} = 3 \cdot \sqrt[3]{3}$$

$$(e) \quad \sqrt[4]{162} = \sqrt[4]{2 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = \sqrt[4]{2 \cdot 3^4} = 3 \cdot \sqrt[4]{2}$$

$$(b) \quad \sqrt[3]{40} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 5} = \sqrt[3]{2^3 \cdot 5} = 2 \cdot \sqrt[3]{5}$$

$$(d) \quad \sqrt[4]{32} = \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = \sqrt[4]{2^4 \cdot 2} = 2 \cdot \sqrt[4]{2}$$

10. Vereinfache durch teilweises Wurzelziehen so weit wie möglich!

$$\begin{aligned} \text{(a)} \quad & 5\sqrt{2} + 7\sqrt{2} - 3\sqrt{2} = \\ & = (5 + 7 - 3) \cdot \sqrt{2} = \\ & = 9 \cdot \sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & 7\sqrt{8} - \sqrt{32} + 5\sqrt{2} = \\ & = 7\sqrt{2^3} - \sqrt{2^5} + 5\sqrt{2} = \\ & = 7 \cdot 2\sqrt{2} - 4\sqrt{2} + 5\sqrt{2} = \\ & = (14 - 4 + 5) \cdot \sqrt{2} = \\ & = 15 \cdot \sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & 3\sqrt[3]{2} - \sqrt[3]{16} + \sqrt[3]{54} = \\ & = 3\sqrt[3]{2} - \sqrt[3]{2^4} + \sqrt[3]{2 \cdot 3^3} = \\ & = 3\sqrt[3]{2} - 2\sqrt[3]{2} + 3\sqrt[3]{2} = \\ & = (3 - 2 + 3) \cdot \sqrt[3]{2} = \\ & = 4 \cdot \sqrt[3]{2} \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad & 6\sqrt[3]{32} - 5\sqrt[3]{108} + 2\sqrt[3]{256} = \\ & = 6\sqrt[3]{2^5} - 5\sqrt[3]{2^2 \cdot 3^3} + 2\sqrt[3]{2^8} = \\ & = 6 \cdot 2\sqrt[3]{2^2} - 5 \cdot 3\sqrt[3]{2^2} + 2 \cdot 2^2\sqrt[3]{2^2} = \\ & = \sqrt[3]{2^2} \cdot (12 - 15 + 8) = \\ & = 5\sqrt[3]{4} \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad & 5\sqrt[3]{5} - 6\sqrt[3]{40} + 3\sqrt[3]{135} = \\ & = 5\sqrt[3]{5} - 6\sqrt[3]{2^3 \cdot 5} + 3\sqrt[3]{3^3 \cdot 5} = \\ & = 5\sqrt[3]{5} - 6 \cdot 2\sqrt[3]{5} + 3 \cdot 3\sqrt[3]{5} = \\ & = \sqrt[3]{5} \cdot (5 - 12 + 9) = \\ & = 2\sqrt[3]{5} \end{aligned}$$

$$\begin{aligned} \text{(f)} \quad & \frac{5\sqrt{27} + 3\sqrt{6} - \sqrt{3}}{2\sqrt{3}} = \\ & = \frac{5\sqrt{3^3} + 3\sqrt{2 \cdot 3} - \sqrt{3}}{2\sqrt{3}} = \\ & = \frac{\sqrt{3} \cdot (5 \cdot 3 + 3\sqrt{2} - 1)}{2\sqrt{3}} = \\ & = \frac{15 + 3\sqrt{2} - 1}{2} = \frac{3\sqrt{2} + 14}{2} = \\ & = \frac{3\sqrt{2}}{2} + 7 \end{aligned}$$

11. Mache den Nenner rational!

$$\text{(a)} \quad \frac{5}{\sqrt{5}} = \frac{5 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{5 \cdot \sqrt{5}}{5} = \sqrt{5}$$

$$\begin{aligned} \text{(b)} \quad & \frac{45}{\sqrt{27}} = \frac{45 \cdot \sqrt{27}}{\sqrt{27} \cdot \sqrt{27}} = \frac{45 \cdot \sqrt{3^3}}{27} = \frac{5 \cdot 3\sqrt{3}}{3} \\ & = 5 \cdot \sqrt{3} \end{aligned}$$

$$\text{(c)} \quad \frac{3}{\sqrt[3]{3}} = \frac{3 \cdot \sqrt[3]{3^2}}{\sqrt[3]{3} \cdot \sqrt[3]{3^2}} = \frac{3 \cdot \sqrt[3]{3^2}}{3} = \sqrt[3]{3^2} = \sqrt[3]{9}$$

$$\text{(d)} \quad \frac{2}{\sqrt[3]{24}} = \frac{2}{\sqrt[3]{2^3 \cdot 3}} = \frac{2}{2 \cdot \sqrt[3]{3}} = \frac{\sqrt[3]{3^2}}{\sqrt[3]{3} \cdot \sqrt[3]{3^2}} = \frac{\sqrt[3]{9}}{3}$$

$$\text{(e)} \quad \frac{16}{\sqrt[4]{8}} = \frac{2^4 \cdot \sqrt[4]{2}}{\sqrt[4]{2^3} \cdot \sqrt[4]{2}} = \frac{2^4 \cdot \sqrt[4]{2}}{2} = 2^3 \cdot \sqrt[4]{2}$$

$$\begin{aligned} \text{(f)} \quad & \sqrt[3]{\frac{2}{45}} = \frac{\sqrt[3]{2}}{\sqrt[3]{3^2 \cdot 5}} = \frac{\sqrt[3]{2} \cdot \sqrt[3]{3} \cdot \sqrt[3]{5^2}}{\sqrt[3]{3^2 \cdot 5} \cdot \sqrt[3]{3} \cdot \sqrt[3]{5^2}} = \\ & = \frac{\sqrt[3]{2 \cdot 3 \cdot 5^2}}{3 \cdot 5} = \frac{\sqrt[3]{150}}{15} \end{aligned}$$

$$\begin{aligned} \text{(g)} \quad & \left(\sqrt[3]{\frac{7}{50}} \right)^2 = \frac{\sqrt[3]{7^2}}{\sqrt[3]{(2 \cdot 5^2)^2}} = \frac{\sqrt[3]{7^2}}{\sqrt[3]{2^2 \cdot 5^4}} = \\ & = \frac{7^{\frac{2}{3}}}{2^{\frac{2}{3}} \cdot 5^{\frac{4}{3}}} = \frac{7^{\frac{2}{3}}}{2^{\frac{2}{3}} \cdot 5 \cdot 5^{\frac{1}{3}}} = \\ & = \frac{7^{\frac{2}{3}} \cdot 2^{\frac{1}{3}} \cdot 5^{\frac{2}{3}}}{2^{\frac{2}{3}} \cdot 5 \cdot 5^{\frac{1}{3}} \cdot 2^{\frac{1}{3}} \cdot 5^{\frac{2}{3}}} = \\ & = \frac{2^{\frac{1}{3}} \cdot 5^{\frac{2}{3}} \cdot 7^{\frac{2}{3}}}{2 \cdot 5 \cdot 5} = \frac{\sqrt[3]{2 \cdot 5^2 \cdot 7^2}}{50} \end{aligned}$$

$$\begin{aligned} \text{(h)} \quad & \frac{4}{2 + \sqrt{2}} = \frac{4 \cdot (2 - \sqrt{2})}{(2 + \sqrt{2}) \cdot (2 - \sqrt{2})} = \\ & = \frac{4 \cdot (2 - \sqrt{2})}{4 - 2} = \\ & = \frac{4 \cdot (2 - \sqrt{2})}{2} = 2 \cdot (2 - \sqrt{2}) = \\ & = 4 - 2 \cdot \sqrt{2} \end{aligned}$$

$$\begin{aligned}
 \text{(i)} \quad \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} &= \frac{(\sqrt{5} + \sqrt{3}) \cdot (\sqrt{5} + \sqrt{3})}{(\sqrt{5} - \sqrt{3}) \cdot (\sqrt{5} + \sqrt{3})} = \\
 &= \frac{(\sqrt{5} + \sqrt{3})^2}{5 - 3} = \frac{5 + 2\sqrt{3}\sqrt{5} + 3}{2} = \\
 &= \frac{8 + 2\sqrt{15}}{2} = 4 + \sqrt{15}
 \end{aligned}$$

$$\begin{aligned}
 \text{(i)} \quad \frac{4\sqrt{3}}{\sqrt{7} - \sqrt{3}} &= \frac{4\sqrt{3} \cdot (\sqrt{7} + \sqrt{3})}{(\sqrt{7} - \sqrt{3}) \cdot (\sqrt{7} + \sqrt{3})} = \\
 &= \frac{4\sqrt{3} \cdot (\sqrt{7} + \sqrt{3})}{7 - 3} = \\
 &= \sqrt{3} \cdot (\sqrt{7} + \sqrt{3}) = \\
 &= \sqrt{21} + 3
 \end{aligned}$$

12. Vereinfache!

$$\begin{aligned}
 \text{(a)} \quad & \sqrt[3]{\sqrt{27x^3y^9}} = \left((3^3x^3y^9)^{\frac{1}{2}} \right)^{\frac{1}{3}} = \\
 &= (3^3x^3y^9)^{\frac{1}{6}} = 3^{\frac{1}{2}}x^{\frac{1}{2}}y^{\frac{3}{2}} = \\
 &= y\sqrt{3xy}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & \sqrt[3]{\sqrt{\frac{25}{49}a^2b^{-4}c^8}} = \left(\left(\frac{5^2}{7^2}a^2b^{-4}c^8 \right)^{\frac{1}{3}} \right)^{\frac{1}{2}} = \\
 &= \left(\frac{5^2}{7^2}a^2b^{-4}c^8 \right)^{\frac{1}{6}} = \frac{5^{\frac{1}{3}}}{7^{\frac{1}{3}}}a^{\frac{1}{3}}b^{-\frac{2}{3}}c^{\frac{4}{3}} = \\
 &= \frac{5^{\frac{1}{3}}a^{\frac{1}{3}}c^{\frac{4}{3}}}{7^{\frac{1}{3}}b^{\frac{2}{3}}} = c\sqrt[3]{\frac{5ac}{7b^2}}
 \end{aligned}$$

13. Ermittle die Lösungsmenge in $\mathbb{Q}$!

(a)

$$\begin{aligned}
 & \sqrt{x+12} + \sqrt{x-3} = \sqrt{x+32} - \sqrt{5+x} \quad |^2 \\
 x+12 + 2 \cdot \sqrt{x+12} \cdot \sqrt{x-3} + x-3 &= x+32 - 2 \cdot \sqrt{x+32} \cdot \sqrt{5+x} + 5+x \\
 2x+9 + 2 \cdot \sqrt{(x+12)(x-3)} &= 2x+37 - 2 \cdot \sqrt{(x+32)(5+x)} \quad | -2x | -9 \\
 2 \cdot \sqrt{x^2-3x+12x-36} &= 28 - 2 \cdot \sqrt{5x+x^2+160+32x} \quad | :2 \\
 \sqrt{x^2+9x-36} &= 14 - \sqrt{x^2+37x+160} \quad |^2 \\
 x^2+9x-36 &= 196 - 28 \cdot \sqrt{x^2+37x+160} + x^2+37x+160 \quad | -196 | -x^2 | -37x | -160 \\
 -28x-392 &= -28\sqrt{x^2+37x+160} \quad | :(-28) \\
 x+14 &= \sqrt{x^2+37x+160} \quad |^2 \\
 x^2+28x+196 &= x^2+37x+160 \quad | -x^2 | -37x | -196 \\
 -9x &= -36 \quad | :(-9) \\
 x &= 4
 \end{aligned}$$

Probe:

$$\begin{aligned}
 T_L(4) &= \sqrt{16} + \sqrt{1} = 4 + 1 = 5 \\
 T_R(4) &= \sqrt{36} - \sqrt{9} = 6 - 3 = 3
 \end{aligned}
 \Rightarrow L = \{ \}$$

(b)

$$\begin{aligned}\sqrt{x+20} &= \sqrt{x} + \sqrt{x-12} \quad |^2 \\ x+20 &= x + 2 \cdot \sqrt{x} \cdot \sqrt{x-12} + x - 12 \\ x+20 &= 2x - 12 + 2 \cdot \sqrt{x(x-12)} \quad | -2x | +12 \\ -x+32 &= 2 \cdot \sqrt{x^2-12x} \quad | :2 \\ 16 - \frac{x}{2} &= \sqrt{x^2-12x} \quad |^2 \\ 256 - 16x + \frac{x^2}{4} &= x^2 - 12x \quad | -x^2 | +12x \\ -\frac{3}{4}x^2 - 4x + 256 &= 0 \quad | \cdot (-4) \\ 3x^2 + 16x - 1024 &= 0 \\ x_{1,2} &= \frac{-16 \pm \sqrt{16^2 - 4 \cdot 3 \cdot (-1024)}}{2 \cdot 3} = \frac{-16 \pm 112}{6} \\ x_1 &= -\frac{64}{3} \quad x_2 = 16\end{aligned}$$

Probe:

$$\begin{aligned}T_L\left(-\frac{64}{3}\right) &= \sqrt{-\frac{64}{3} + 20} = \sqrt{-\frac{4}{3}} \\ T_R\left(-\frac{64}{3}\right) &= \sqrt{-\frac{64}{3}} + \sqrt{-\frac{64}{3} - 12}\end{aligned} \left. \begin{array}{l} \\ \end{array} \right\} \Rightarrow \begin{array}{l} \text{Wurzeln negativ, d.h. nicht definiert,} \\ \text{somit ist } -\frac{64}{3} \text{ keine Lösung} \end{array}$$
$$\left. \begin{array}{l} T_L(16) = \sqrt{16 + 20} = \sqrt{36} = 6 \\ T_R(16) = \sqrt{16} + \sqrt{16 - 12} = 4 + 2 = 6 \end{array} \right\} \Rightarrow L = \{16\}$$

(c)

$$\begin{aligned}\sqrt{4x-9} + \sqrt{6x-7} &= \sqrt{5x-3} + \sqrt{5x-13} \quad |^2 \\ 4x-9 + 2 \cdot \sqrt{4x-9} \cdot \sqrt{6x-7} + 6x-7 &= 5x-3 + 2 \cdot \sqrt{5x-3} \cdot \sqrt{5x-13} + 5x-13 \\ 2 \cdot \sqrt{4x-9} \cdot \sqrt{6x-7} + 10x-16 &= 2 \cdot \sqrt{5x-3} \cdot \sqrt{5x-13} + 10x-16 \quad | -10x | +16 : 2 \\ \sqrt{4x-9} \cdot \sqrt{6x-7} &= \sqrt{5x-3} \cdot \sqrt{5x-13} \quad |^2 \\ (4x-9) \cdot (6x-7) &= (5x-3) \cdot (5x-13) \\ 24x^2 - 28x - 54x + 63 &= 25x^2 - 65x - 15x + 39 \\ 24x^2 - 82x + 63 &= 25x^2 - 80x + 39 \quad | -24x^2 | +82x | -63 \\ 0 &= x^2 + 2x - 24 \\ x_{1,2} &= -1 \pm \sqrt{1^2 + 24} = -1 \pm 5 \\ x_1 &= -6 \quad x_2 = 4\end{aligned}$$

Probe:

$$\begin{aligned}T_L(-6) &= \sqrt{4 \cdot (-6) - 9} + \sqrt{6 \cdot (-6) - 7} = \sqrt{-33} + \sqrt{-43} \\ T_R(-6) &= \sqrt{5 \cdot (-6) - 3} + \sqrt{5 \cdot (-6) - 13} = \sqrt{-33} + \sqrt{-43} \\ &\Rightarrow \text{Wurzeln negativ, d.h. nicht definiert,} \\ &\quad \text{somit ist -6 keine Lösung} \\ T_L(4) &= \sqrt{4 \cdot 4 - 9} + \sqrt{6 \cdot 4 - 7} = \sqrt{7} + \sqrt{17} \\ T_R(4) &= \sqrt{5 \cdot 4 - 3} + \sqrt{5 \cdot 4 - 13} = \sqrt{17} + \sqrt{7} \\ &\Rightarrow L = \{4\}\end{aligned}$$

(d)

$$\begin{aligned}2\sqrt{3x+1} &= \sqrt{x+4} + x \quad |^2 \\ 4 \cdot (3x+1) &= x+4 + 2 \cdot x \cdot \sqrt{x+4} + x^2 \\ 12x+4 &= x^2 + x + 4 + 2x\sqrt{x+4} \quad | -x^2 | -x | -4 \\ -x^2 + 11x &= 2x\sqrt{x+4} \quad |^2 \\ x^4 - 22x^3 + 121x^2 &= 4x^2(x+4) \\ x^4 - 22x^3 + 121x^2 &= 4x^3 + 16x^2 \quad | -4x^3 | -16x^2 \\ x^4 - 26x^3 + 105x^2 &= 0 \\ x^2(x^2 - 26x + 105) &= 0 \\ x_{2,3} &= 13 \pm \sqrt{13^2 - 105} = 13 \pm 8 \\ x_1 &= 0 \quad x_2 = 5 \quad x_3 = 21\end{aligned}$$

Probe:

$$\begin{aligned}T_L(0) &= 2\sqrt{3 \cdot 0 + 1} = 2\sqrt{1} = 2 \\ T_R(0) &= \sqrt{0 + 4} + 0 = 2 \\ T_L(5) &= 2\sqrt{3 \cdot 5 + 1} = 2\sqrt{16} = 8 \\ T_R(5) &= \sqrt{5 + 4} + 5 = 3 + 5 = 8 \\ &\Rightarrow \text{w.A.} \\ T_L(21) &= 2\sqrt{3 \cdot 21 + 1} = 2\sqrt{64} = 16 \\ T_R(21) &= \sqrt{21 + 4} + 21 = 5 + 21 = 26 \\ &\Rightarrow \text{f.A.} \\ &\Rightarrow L = \{0; 5\}\end{aligned}$$

(e)

$$\begin{aligned}\sqrt{4+2x} &= 1 + \sqrt{3+x} \quad |^2 \\ 4+2x &= 1 + 2\sqrt{3+x} + 3+x \\ 4+2x &= 2\sqrt{3+x} + 4+x \quad | -4 | -x \\ x &= 2\sqrt{3+x} \quad | :2 \\ \frac{1}{2}x &= \sqrt{3+x} \quad |^2 \\ \frac{1}{4}x^2 &= 3+x \quad | -3 | -x \\ \frac{1}{4}x^2 - x - 3 &= 0 \quad | \cdot 4 \\ x^2 - 4x - 12 &= 0 \\ x_{1,2} &= 2 \pm \sqrt{4+12} = 2 \pm 4 \\ x_1 &= -2 \quad x_2 = 6\end{aligned}$$

Probe:

$$\begin{aligned}T_L(-2) &= \sqrt{4+2(-2)} = \sqrt{0} = 0 \\ T_R(-2) &= 1 + \sqrt{3-2} = 1 + \sqrt{1} = 2\end{aligned} \left. \vphantom{\begin{aligned}T_L(-2) &= \sqrt{4+2(-2)} = \sqrt{0} = 0 \\ T_R(-2) &= 1 + \sqrt{3-2} = 1 + \sqrt{1} = 2\end{aligned}} \right\} f.A.$$
$$\begin{aligned}T_L(6) &= \sqrt{4+12} = \sqrt{16} = 4 \\ T_R(6) &= 1 + \sqrt{3+6} = 1 + \sqrt{9} = 4\end{aligned} \left. \vphantom{\begin{aligned}T_L(6) &= \sqrt{4+12} = \sqrt{16} = 4 \\ T_R(6) &= 1 + \sqrt{3+6} = 1 + \sqrt{9} = 4\end{aligned}} \right\} w.A.$$
$$\Rightarrow L = \{6\}$$

(f)

$$\begin{aligned}\sqrt{4x-3} &= \sqrt{25x-11} - \sqrt{9x-2} \quad |^2 \\ 4x-3 &= 25x-11 - 2 \cdot \sqrt{25x-11} \cdot \sqrt{9x-2} + 9x-2 \\ 4x-3 &= -2 \cdot \sqrt{25x-11} \cdot \sqrt{9x-2} + 34x-13 \quad | -34x | +13 \\ -30x+10 &= -2 \cdot \sqrt{(25x-11) \cdot (9x-2)} \quad | :(-2) \\ 15x-5 &= \sqrt{225x^2-50x-99x+22} \quad |^2 \\ 225x^2-150x+25 &= 225x^2-149x+22 \quad | -225x^2 | +150x | -25 \\ 0 &= x-3 \quad | +3 \\ x &= 3\end{aligned}$$

Probe:

$$\begin{aligned}T_L(3) &= \sqrt{12-3} = \sqrt{9} = 3 \\ T_R(3) &= \sqrt{75-11} - \sqrt{27-2} = \sqrt{64} - \sqrt{25} = 8 - 5 = 3\end{aligned}$$