

1. Vereinfache mit Hilfe der Rechengesetze für Potenzen!

$$(a) \left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5}$$

$$(b) (-3)^5 = -3^5$$

$$(c) 2^4 \cdot 3^4 = (2 \cdot 3)^4 = 6^4$$

$$(d) \frac{2^5}{4^5} = \left(\frac{2}{4}\right)^5 = \left(\frac{1}{2}\right)^5$$

$$(e) (2^3)^4 = 2^{3 \cdot 4} = 2^{12}$$

2. Vereinfache mit Hilfe der Rechengesetze für Potenzen!

$$(a) a^3 \cdot a = a^{3+1} = a^4$$

$$(b) b^5 \cdot b^3 = b^{5+3} = b^8$$

$$(c) (-c)^2 \cdot (-c)^5 = (-c)^{2+5} = (-c)^7 = -c^7$$

$$(d) \frac{d^5}{d} = d^{5-1} = d^4$$

$$(e) (-e)^4 \cdot (-e)^2 = (-e)^{4+2} = (-e)^6 = e^6$$

3. Berechne ohne Taschenrechner!

$$(a) 5^7 \cdot 5^{-3} = 5^{7-3} = 5^4$$

$$(b) 3^{-2} \cdot 6^{-2} = (3 \cdot 6)^{-2} = 18^{-2} = \frac{1}{18^2}$$

$$(c) \frac{7^{-8}}{7^{-3}} = 7^{-8-(-3)} = 7^{-5} = \frac{1}{7^5}$$

$$(d) \frac{3^{-5}}{12^{-5}} = \left(\frac{3}{12}\right)^{-5} = \left(\frac{1}{4}\right)^{-5} = 4^5$$

$$(e) (3^{-2})^3 = 3^{-2 \cdot 3} = 3^{-6} = \frac{1}{3^6}$$

4. Vereinfache auf eine möglichst kleine Basis mit positivem Exponenten!

$$(a) 8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^{3 \cdot \frac{2}{3}} = 2^2$$

$$(b) 16^{\frac{3}{4}} = (2^4)^{\frac{3}{4}} = 2^{4 \cdot \frac{3}{4}} = 2^3$$

$$(c) 7^{\frac{3}{8}} = \sqrt[8]{7^3}$$

$$(d) 4^{-\frac{5}{2}} = (2^2)^{-\frac{5}{2}} = 2^{-2 \cdot \frac{5}{2}} = 2^{-5} = \frac{1}{2^5}$$

$$(e) 27^{-\frac{4}{3}} = (3^3)^{-\frac{4}{3}} = 3^{-3 \cdot \frac{4}{3}} = 3^{-4} = \frac{1}{3^4}$$

5. Vereinfache! Achtung, hier sind die Bruch-Exponenten größer als 1!

$$(a) 2^{\frac{7}{3}} = 2^{\frac{6}{3}} \cdot 2^{\frac{1}{3}} = 2^2 \cdot \sqrt[3]{2}$$

$$(b) 6^{\frac{5}{3}} = 6^{\frac{3}{3}} \cdot 6^{\frac{2}{3}} = 6 \cdot \sqrt[3]{6^2}$$

$$(c) 4^{\frac{5}{2}} = 4^{\frac{4}{2}} \cdot 4^{\frac{1}{2}} = 4^2 \cdot \sqrt{4}$$

$$(d) 25^{\frac{7}{4}} = (5^2)^{\frac{7}{4}} = 5^{2 \cdot \frac{7}{4}} = 5^{\frac{7}{2}} = 5^{\frac{6}{2}} \cdot 5^{\frac{1}{2}} = 5^3 \cdot \sqrt{5}$$

$$(e) 27^{\frac{3}{2}} = (3^3)^{\frac{3}{2}} = 3^{3 \cdot \frac{3}{2}} = 3^{\frac{9}{2}} = 3^{\frac{8}{2}} \cdot 3^{\frac{1}{2}} = 3^4 \cdot \sqrt{3}$$

6. Vereinfache so weit wie möglich!

$$(a) \left(\frac{1}{5^{-\frac{2}{3}}}\right)^2 = \left(5^{\frac{2}{3}}\right)^2 = 5^{\frac{2}{3} \cdot 2} = 5^{\frac{4}{3}} = 5^{\frac{3}{3}} \cdot 5^{\frac{1}{3}} = 5 \cdot \sqrt[3]{5}$$

$$(b) \left(\frac{5^{\frac{2}{3}}}{5^{-\frac{1}{2}}}\right)^3 = \left(5^{\frac{2}{3} - (-\frac{1}{2})}\right)^3 = \left(5^{\frac{4}{6} + \frac{3}{6}}\right)^3 = \left(5^{\frac{7}{6}}\right)^3 = 5^{\frac{7}{6} \cdot 3} = 5^{\frac{7}{2}} = 5^{\frac{6}{2}} \cdot 5^{\frac{1}{2}} = 5^3 \cdot \sqrt{5}$$

$$(c) \left(\frac{7^{-\frac{3}{2}}}{7^{\frac{2}{3}}}\right)^6 = \left(7^{-\frac{3}{2} - \frac{2}{3}}\right)^6 = \left(7^{-\frac{9}{6} - \frac{4}{6}}\right)^6 = \left(7^{-\frac{13}{6}}\right)^6 = 7^{-\frac{13}{6} \cdot 6} = 7^{-13} = \frac{1}{7^{13}}$$

(d)

$$(e) \quad \frac{4^{\frac{1}{3}} \cdot 2^{\frac{3}{4}}}{8^{\frac{1}{2}}} = \frac{(2^2)^{\frac{1}{3}} \cdot 2^{\frac{3}{4}}}{(2^3)^{\frac{1}{2}}} = \frac{2^{\frac{2}{3}} \cdot 2^{\frac{3}{4}}}{2^{\frac{3}{2}}} = \frac{2^{\frac{2}{3}} \cdot 2^{\frac{3}{4}}}{2^{\frac{3}{2}}} =$$

$$= 2^{\frac{2}{3} + \frac{3}{4} - \frac{3}{2}} = 2^{\frac{8}{12} + \frac{9}{12} - \frac{18}{12}} = 2^{-\frac{1}{12}} = \frac{1}{\sqrt[12]{2}}$$

$$\left(8^{-\frac{5}{6}} \cdot 4^{\frac{3}{4}}\right)^{-2} = \left((2^3)^{-\frac{5}{6}} \cdot (2^2)^{\frac{3}{4}}\right)^{-2} =$$

$$= \left(\left(2^{-\frac{5}{6}}\right) \cdot \left(2^{\frac{3}{4}}\right)\right)^{-2} =$$

$$= \left(2^{-\frac{5}{2}} \cdot 2^{\frac{3}{2}}\right)^{-2} = \left(2^{-\frac{5}{2} + \frac{3}{2}}\right)^{-2} =$$

$$= 2^{\frac{2}{2}(-2)} = 2^2$$

7. Vereinfache und stelle das Ergebnis mit positiven Hochzahlen dar!

$$(a) \quad \left[\left(\frac{-7ax}{3b^2y}\right)^4 : \left(\frac{7ax}{12b^3y^2}\right)^3\right] \cdot \left(\frac{by}{2ax}\right)^5 =$$

$$= \frac{7^4 a^4 x^4}{3^4 b^8 y^4} : \frac{7^3 a^3 x^3}{(2^2 \cdot 3)^3 b^9 y^6} \cdot \frac{b^5 y^5}{2^5 a^5 x^5} =$$

$$= \frac{7^4 a^4 x^4}{3^4 b^8 y^4} \cdot \frac{2^6 3^3 b^9 y^6}{7^3 a^3 x^3} \cdot \frac{b^5 y^5}{2^5 a^5 x^5} =$$

$$= \frac{2^6 3^3 7^4 a^4 b^{14} x^4 y^{11}}{2^5 3^4 7^3 a^8 b^8 x^8 y^4} =$$

$$= \frac{2 \cdot 7 b^6 y^7}{3 a^4 x^4} = \frac{14 b^6 y^7}{3 a^4 x^4}$$

$$(b) \quad \left(\frac{c^{-3}}{a^{-2}b^{-3}}\right)^2 : \left(\frac{a^{-3}b}{c}\right)^{-4} \cdot \frac{(a^{-1}b)^{-1}}{(c^{-1})^2} =$$

$$= \frac{c^{-6}}{a^{-4}b^{-6}} : \frac{a^{12}b^{-4}}{c^{-4}} \cdot \frac{a^1 b^{-1}}{c^{-2}} =$$

$$= \frac{c^{-6}}{a^{-4}b^{-6}} \cdot \frac{c^{-4}}{a^{12}b^{-4}} \cdot \frac{a^1 b^{-1}}{c^{-2}} =$$

$$= \frac{a^4 b^6}{c^6} \cdot \frac{b^4}{a^{12} c^4} \cdot \frac{a c^2}{b} =$$

$$= \frac{a^5 b^{10} c^2}{a^{12} b c^{10}} = \frac{b^9}{a^7 c^8}$$

$$(c) \quad \left[\left(-\frac{4ac^{-1}}{3b^{-3}d}\right)^{-2} : \left(\frac{(a-b)^2}{(2ab^2)^3}\right)\right] \cdot \frac{c^{-1}d^{-3}}{3(a-b)^{-1}} =$$

$$= -\frac{(2^2)^{-2} a^{-2} c^2}{3^{-2} b^6 d^{-2}} \cdot \frac{2^3 a^3 b^6}{(a-b)^2} \cdot \frac{c^{-1} d^{-3}}{3(a-b)^{-1}} =$$

$$= -\frac{3^2 c^2 d^2}{2^4 a^2 b^6} \cdot \frac{2^3 a^3 b^6}{(a-b)^2} \cdot \frac{(a-b)}{3 c d^3} =$$

$$= -\frac{3c}{2} \cdot \frac{a}{(a-b)} \cdot \frac{1}{d} =$$

$$= -\frac{3ac}{2d(a-b)}$$

$$(d) \quad \left[\frac{3y^{-3}}{2x(a-b)^{-1}}\right]^{-3} : \left[\frac{(2x^{-1}y^3)^2}{(a-b)^4} : \frac{9x^{-5}y^{-3}}{(a+b)^{-1}}\right]$$

$$= \frac{3^{-3} y^9}{2^{-3} x^{-3} (a-b)^3} : \left[\frac{2^2 x^{-2} y^6}{(a-b)^4} \cdot \frac{(a+b)^{-1}}{3^2 x^{-5} y^{-3}}\right]$$

$$= \frac{2^3 x^3 y^9}{3^3 (a-b)^3} : \left[\frac{2^2 y^6}{x^2 (a-b)^4} \cdot \frac{x^5 y^3}{3^2 (a+b)^1}\right] =$$

$$= \frac{2^3 x^3 y^9}{3^3 (a-b)^3} \cdot \frac{2^2 x^3 y^9}{3^2 (a-b)^4 (a+b)^1} =$$

$$= \frac{2^3 x^3 y^9}{3^3 (a-b)^3} \cdot \frac{3^2 (a-b)^4 (a+b)^1}{2^2 x^3 y^9} =$$

$$= \frac{2(a-b)(a+b)}{3} = \frac{2(a^2 - b^2)}{3}$$

$$(e) \quad \left[\frac{2^3 x^{-4} y^2}{9^{-1} a^{-5} b^3}\right]^{-2} : \left[-\frac{3^2 x^{-2} y^5}{3 a^{-2} b^2}\right]^{-3} =$$

$$= \frac{2^{-6} x^8 y^{-4}}{(3^2)^2 a^{10} b^{-6}} : \left(-\frac{3^{-6} x^6 y^{-15}}{3^{-3} a^6 b^{-6}}\right) =$$

$$= -\frac{x^8 b^6}{2^6 3^4 a^{10} y^4} : \frac{3^3 b^6 x^6}{3^6 a^6 y^{15}} =$$

$$= -\frac{x^8 b^6}{2^6 3^4 a^{10} y^4} \cdot \frac{3^6 a^6 y^{15}}{3^3 b^6 x^6} =$$

$$= -\frac{3^6 a^6 b^6 x^8 y^{15}}{2^6 3^7 a^{10} b^6 x^6 y^4} =$$

$$= -\frac{x^2 y^{11}}{192 a^4}$$

$$(f) \quad \left(\frac{3^2 a}{4b}\right)^{-4} : \left(\frac{16 \cdot 3^{-1} a^{-2}}{-b^{-2}}\right)^2 =$$

$$= \frac{4^4 b^4}{3^8 a^4} : \frac{2^8 b^4}{3^2 a^4} =$$

$$= \frac{2^8 b^4 3^2 a^4}{3^8 a^4 2^8 b^4} =$$

$$= \frac{1}{3^6}$$