**13.** Berechne ohne Taschenrechner!

(a) $^{3}log\,\frac{1}{\sqrt[5]{9}} = x$

$3^x = 9^{-\frac{1}{5}}$

$3^x = \left(3^2\right)^{-\frac{1}{5}} = 3^{-\frac{2}{5}}$

$x = -\frac{2}{5}$

(b) $^{5}log\,\sqrt[4]{125} = x$

$5^x = \left(5^3\right)^{\frac{1}{4}}$

$5^x = 5^{\frac{3}{4}}$

$x = \frac{3}{4}$

(c) $\lg\sqrt{0{,}1} = x$

$^{10}\log\left(\frac{1}{10}\right)^{\frac{1}{2}} = x$

$10^x = 10^{-\frac{1}{2}}$

$x = -\frac{1}{2}$

(d) $\lg\frac{1}{\sqrt[3]{100}} = x$

$^{10}\log 10^{-\frac{2}{3}} = x$

$10^x = 10^{-\frac{2}{3}}$

$x = -\frac{2}{3}$

(e) $\ln\sqrt[5]{\frac{1}{e^2}} = x$

$^{e}\log e^{-\frac{2}{5}} = x$

$e^x = e^{-\frac{2}{5}}$

$x = -\frac{2}{5}$

(f) $\ln\frac{e}{\sqrt{e}} = x$

$^{e}\log e^{1-\frac{1}{2}} = x$

$^{e}\log e^{\frac{1}{2}} = x$

$e^x = e^{\frac{1}{2}}$

$x = \frac{1}{2}$

(g) $^{a}log\,1 = x$

$a^x = 1$

$x = 0$

(h) $^{2}log - 4 = x$

$2^x = -4$

nicht möglich!

(i) $\ln 1 = x$

$^{e}\log 1 = x$

$e^x = 1$

$x = 0$

(j) $\ln e = x$

$^{e}\log e = x$

$e^x = e$

$x = 1$

**14.** Berechne x!

(a) ${}^{x}\log 9 = -2$

$x^{-2} = 9$

$\frac{1}{x^2} = 9 \quad |\cdot x^2 \,|:9$

$x^2 = \frac{1}{9}$

$x = \frac{1}{3}$ (nur positiv, da log nur für $\mathbf{R}^+$ definiert ist)

(b) ${}^{x}\log a^4 = 2$

$x^2 = a^4 \quad |\sqrt{\ }$

$x = a^2$

(c) ${}^{x}\log 27 = -\frac{3}{4}$

$x^{-\frac{3}{4}} = 27$

$\frac{1}{\sqrt[4]{x^3}} = 27 \quad |^4|\sqrt[3]{\ }$

$\frac{1}{x} = 3^4 \quad |\cdot x|:3^4$

$x = \frac{1}{81}$

(d) ${}^{x}\log 5 = 2$

$x^2 = 5 \quad |\sqrt{\ }$

$x = \sqrt{5}$

(e) ${}^{4}\log x = -1$

$4^{-1} = x$

$x = \frac{1}{4}$

(f) $\lg x = -\frac{1}{4}$

${}^{10}\log x = -\frac{1}{4}$

$10^{-\frac{1}{4}} = x$

$x = \frac{1}{\sqrt[4]{10}}$

(g) ${}^{a}\log x = a$

$x = a^a$

**15.** Berechne!

(a) $lg\left(\underbrace{lg\,10}_{1}\right) = lg\,1 = 0$

${}^{10}\log 10 = x$

$10^x = 10$

$x = 1$

${}^{10}\log 1 = y$

$10^y = 1$

$y = 0$

(b) $lg\left(\underbrace{ln\,e^{10}}_{10}\right) = lg\,10 = 1$

${}^{e}\log e^{10} = x$

$e^x = e^{10}$

$x = 10$

${}^{10}\log 10 = y$

$10^y = 10$

$y = 1$

(c) $ln\left(\underbrace{ln\,e}_{1}\right) = ln\,1 = 0$

${}^{e}\log e = x$

$e^x = e$

$x = 1$

${}^{e}\log 1 = y$

$e^y = 1$

$y = 0$

(d) $ln\left(\underbrace{lg\,10^e}_{e}\right) = ln\,e = 1$

${}^{10}\log 10^e = x$

$10^x = 10^e$

$x = e$

${}^{e}\log e = y$

$e^y = e$

$y = 1$

**16.** Zerlege mit Hilfe der Logarithmusregeln!

(a)
$$\log\left(x^2\sqrt{\frac{y}{\sqrt[3]{z}}}\right)=\log x^2+\log\left(\frac{y}{\sqrt[3]{z}}\right)^{\frac{1}{2}}=$$
$$=2\log x+\frac{1}{2}\log\left(\frac{y}{\sqrt[3]{z}}\right)=$$
$$=2\log x+\frac{1}{2}\left(\log y-\frac{1}{3}\log z\right)=$$
$$=2\log x+\frac{1}{2}\log y-\frac{1}{6}\log z$$

(b)
$$\log\frac{\sqrt[4]{2a}\cdot\sqrt[3]{b^2}}{4b\cdot\sqrt{a}}=\log\sqrt[4]{2a}+\log\sqrt[3]{b^2}-\log 4-\log b-\log\sqrt{a}=$$
$$=\frac{1}{4}\log 2a+\frac{1}{3}\log b^2-\log 2^2-\log b-\frac{1}{2}\log a=$$
$$=\frac{1}{4}\log 2+\frac{1}{4}\log a+\frac{2}{3}\log b-2\log 2-\log b-\frac{1}{2}\log a=$$
$$=-\frac{7}{4}\log 2-\frac{1}{4}\log a-\frac{1}{3}\log b$$

(c)
$$\log\left(4x^5\sqrt[3]{\frac{x^2-a^2}{2x^3\sqrt{a}}}\right)=\log 4+\log x^5+\log\left(\frac{x^2-a^2}{2x^3\sqrt{a}}\right)^{\frac{1}{3}}=$$
$$=\log 2^2+\log x^5+\frac{1}{3}\log\left(\frac{x^2-a^2}{2x^3\sqrt{a}}\right)=$$
$$=2\log 2+5\log x+\frac{1}{3}\left(\log\left(x^2-a^2\right)-\log\left(2x^3\sqrt{a}\right)\right)=$$
$$=2\log 2+5\log x+\frac{1}{3}\left(\log\left((x+a)(x-a)\right)-\log 2-\log x^3-\log\sqrt{a}\right)=$$
$$=2\log 2+5\log x+\frac{1}{3}\left(\log(x+a)+\log(x-a)-\log 2-3\log x-\frac{1}{2}\log a\right)=$$
$$=2\log 2+5\log x+\frac{1}{3}\log(x+a)+\frac{1}{3}\log(x-a)-\frac{1}{3}\log 2-\log x-\frac{1}{6}\log a=$$
$$=\frac{5}{3}\log 2+4\log x+\frac{1}{3}\log(x+a)+\frac{1}{3}\log(x-a)-\frac{1}{6}\log a$$

**17.** Stelle als Logarithmus eines einzigen Terms dar!

(a)
$$\frac{1}{3}[5\log x + 2\log(x-y)] - \log x - \frac{1}{2}\log y =$$
$$= \frac{1}{3}[\log x^5 + \log(x-y)^2] - \log x - \log\sqrt{y} =$$
$$= \frac{1}{3}[\log(x^5 \cdot (x-y)^2)] - \log\left(x \cdot \sqrt{y}\right) =$$
$$= \log \frac{\sqrt[3]{x^5 \cdot (x-y)^2}}{x \cdot \sqrt{y}}$$

(b)
$$\frac{1}{2}\left[8\log x + \frac{3}{4}\left(5\log x - \frac{1}{3}\log c + 4\log a + \log b - \frac{2}{3}\log 2\right)\right] =$$
$$= \frac{1}{2}\left[\log x^8 + \frac{3}{4}\left(\log x^5 - \log\sqrt[3]{c} + \log a^4 + \log b - \log\sqrt[3]{2^2}\right)\right] =$$
$$= \frac{1}{2}\left[\log x^8 + \frac{3}{4}\log\frac{a^4 b x^5}{\sqrt[3]{2^2}\sqrt[3]{c}}\right] = \frac{1}{2}\left[\log x^8 + \log\sqrt[4]{\left(\frac{a^4 b x^5}{\sqrt[3]{2^2}\sqrt[3]{c}}\right)^3}\right] =$$
$$= \frac{1}{2}\left[\log x^8 \sqrt[4]{\left(\frac{a^4 b x^5}{\sqrt[3]{2^2}\sqrt[3]{c}}\right)^3}\right] =$$
$$= \log\sqrt{x^8 \sqrt[4]{\left(\frac{a^4 b x^5}{\sqrt[3]{2^2}\sqrt[3]{c}}\right)^3}}$$

(c)
$$3\log x - \frac{2}{3}[4\log y + 3\log z] + \frac{1}{2}\left[\frac{1}{3}\log x - \log y\right] =$$
$$= \log x^3 - \frac{2}{3}[\log y^4 + \log z^3] + \frac{1}{2}\left[\log\sqrt[3]{x} - \log y\right] =$$
$$= \log x^3 - \frac{2}{3}\log y^4 z^3 + \frac{1}{2}\log\frac{\sqrt[3]{x}}{y} =$$
$$= \log x^3 - \log\sqrt[3]{(y^4 z^3)^2} + \log\sqrt{\frac{\sqrt[3]{x}}{y}} =$$
$$= \log \frac{x^3 \sqrt{\frac{\sqrt[3]{x}}{y}}}{\sqrt[3]{(y^4 z^3)^2}}$$

(d)
$$2\log x + \frac{1}{3}\log y + \log(1-x^2) - 4\log x + \frac{1}{2}\log x - [\log(1-x) + \log(1+x) - 2\log x]$$
$$=$$
$$= \log x^2 + \log\sqrt[3]{y} + \log(1-x^2) - \log x^4 + \log\sqrt{x} - \log(1-x) - \log(1+x) + \log x^2 =$$
$$= \log \frac{x^2 \cdot \sqrt[3]{y} \cdot (1-x^2) \cdot \sqrt{x} \cdot x^2}{x^4 \cdot (1-x) \cdot (1+x)} =$$
$$= \log \frac{\sqrt[3]{y} \cdot (1-x^2) \cdot \sqrt{x}}{(1-x^2)} =$$
$$= \log \sqrt{x}\sqrt[3]{y}$$